

UG-C-2284

**BMS-21X/
BMC-21X**

**U.G. DEGREE EXAMINATION –
DECEMBER, 2023.**

Mathematics

Second Year

GROUPS AND RINGS

Time : 3 hours

Maximum marks : 70

PART A — ($3 \times 3 = 9$ marks)

**Answer any THREE questions out of Five questions
in 100 words.**

All questions carry equal marks.

1. Write a note on equivalence relation.
2. Show that in a group G , $x^2 = x$ iff $x = e$.
3. Define isomorphism of a group.
4. Write a short note on maximal ideal.
5. What is meant by euclidean domain?

PART B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Five questions
in 200 words.

All questions carry equal marks.

6. Let $f : A \rightarrow B$, $g : B \rightarrow C$ be bijection and then prove that $g \circ f : A \rightarrow C$ is also a bijection.
7. Show that a non-empty subset H of a group G is a subgroup of G , iff $a, b \in H \Rightarrow a b^{-1} \in H$.
8. Let H and K be two finite subgroups of a group G , then prove that $|H K| = \frac{|H||K|}{|H \cap K|}$.
9. Show that the intersection of two subrings of a ring R is a subring of R .
10. Show that the field of complex number is not an ordered field.

PART C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions out of Seven questions
in 500 words.

All questions carry equal marks.

11. Show that any partition of a set S determines an equivalence relation ρ such that the members of the partition are precisely the equivalence classes define by ρ .

12. Show that the union of two subgroup of a group G is a subgroup, if one is contained in the other.
 13. State and prove Lagrange's Theorem.
 14. State and prove Fundamental theorem of homomorphism of a group.
 15. Let R and R' be rings and $f: R \rightarrow R'$ be an isomorphism. Then prove that the following:
 - (a) R is commutative $\Rightarrow R'$ is commutative
 - (b) R is an ring with identity $\Rightarrow R'$ is a ring with identity
 - (c) R is an integral domain $\Rightarrow R'$ is an integral domain
 - (d) R is a field $\Rightarrow R'$ is a field.
 16. State and prove Cayley's theorem.
 17. Prove that any euclidean domain R is a unique factorization domain.
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**U.G. DEGREE EXAMINATION –
DECEMBER, 2023.**

Mathematics

Second Year

**CLASSICAL ALGEBRA AND NUMERICAL
METHODS**

Time : 3 hours

Maximum marks : 70

PART A — ($3 \times 3 = 9$ marks)

Answer any **THREE** questions out of Five questions in
100 words.

All questions carry equal marks.

1. Prove that

$$\frac{a-x}{a} + \frac{1}{2} \left(\frac{a-x}{a} \right)^2 + \frac{1}{3} \left(\frac{a-x}{a} \right)^3 + \dots = \log a - \log x .$$

2. If α and β are the roots of $2x^2 + 3x + 5 = 0$, find
 $\alpha + \beta, \alpha\beta$.

3. What is the condition for the convergence of the
iterative method for solving $x = \phi(x)$?

4. Prove that:
- (a) $E = 1 + \Delta$
- (b) $E = (1 - \nabla)^{-1}$
5. Given $u_0 = 1, u_1 = 15, u_2 = 57$ find $\frac{dy}{dx}$ at $x = 2$.

PART B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Sum the series $1 - \frac{1}{4} + \frac{1.3}{4.8} - \frac{1.3.5}{4.8.12} + \dots \infty$.
7. Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in A.P.
8. Find the value of $\sqrt{5}$ by Newton-Raphson method.
9. Use Lagrange's interpolation formula to fit a polynomial to the data and find the value of y when $x = 2$
- | | | | | |
|-----|-----|---|---|----|
| x | 0 | 1 | 3 | 4 |
| y | -12 | 0 | 6 | 12 |
10. Solve $y' = x + y$ given $y(1) = 0$ and get $y(1.1), h = 0.1$ by Taylor's series method.

PART C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions out of Seven questions in
500 words.

All questions carry equal marks.

11. Prove that $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.
12. Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.
13. Solve the system of equations by Gauss Jordan method
- $$2x + 3y - z = 5.$$
- $$4x + 4y - 3z = 3$$
- $$2x - 3y + 2z = 2$$
14. Using Laplace-Everett's formula find $\log 337.5$ given that
- | | | | | | | |
|----------|-------|-------|-------|-------|-------|-------|
| x | 310 | 320 | 330 | 340 | 350 | 360 |
| $\log x$ | 2.491 | 2.505 | 2.518 | 2.531 | 2.544 | 2.556 |
15. By dividing the range into six equal parts, evaluate $\int_0^6 \frac{1}{1+x} dx$ using Trapezoidal rule, Simpson's $\frac{1}{3}^{rd}$ rule and Simpson's $\frac{3}{8}^{th}$ rule.

16. (a) Find the value of x when $y=7$ by Lagrange's Interpolation formula.

$$\begin{array}{cccc} x & 1 & 3 & 4 \\ y & 4 & 12 & 19 \end{array}$$

- (b) Express $3x^3 - 2x^2 + 7x - 6$ in factorial polynomials and get their successive forward differences taking $h=1$.
17. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$ find the value of
- (a) $\Sigma \alpha^2$ (b) $\Sigma \alpha^3$
- (c) $\Sigma \alpha^2 \beta$ (d) $\Sigma \alpha^2 \beta^2$
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